Discriminant and the Nature of the Roots for Quadratic Equations (Parabolas)

The Discriminant is an expression that determines the nature of the roots of a quadratic equation, and it can help show us the preferable method of solving a particular quadratic equation.
### The Discriminant: $b^2 - 4ac$

<table>
<thead>
<tr>
<th>Nature of the roots</th>
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</tr>
</thead>
<tbody>
<tr>
<td>zero (b^2 - 4ac)</td>
<td>real, rational, equal</td>
</tr>
<tr>
<td>Pos &amp; (\frac{\sqrt{b^2 - 4ac}}{2a}) perfect square</td>
<td>real, rational, unequal</td>
</tr>
<tr>
<td>Pos &amp; not perfect square</td>
<td>real, irrational, unequal</td>
</tr>
<tr>
<td>negative (b^2 - 4ac)</td>
<td>imaginary</td>
</tr>
</tbody>
</table>

- Positive: \(b^2 - 4ac > 0\)
- Zero: \(b^2 - 4ac = 0\)
- Negative: \(b^2 - 4ac < 0\)

**Diagram:**

- Positive roots: \(x_1, x_2\) \(\left(-\frac{b}{2a}, \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}\right)\)
- Negative roots: \(x_1, x_2\) \(\left(-\frac{b}{2a}, \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}\right)\)
- Zero root: \(x_1, x_2\) \(\left(-\frac{b}{2a}, \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}\right)\)
- Imaginary roots: \(x_1, x_2\) \(\left(-\frac{b}{2a}, \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}\right)\)
Ex: Given $3x^2 - 2x = 4$, which best describes the nature of the roots?

Set equal to zero:

$$3x^2 - 2x - 4 = 0$$

$$a = 3 \quad b = -2 \quad c = -4$$

1) **Unequal, imaginary roots**

2) **Real, unequal, irrational roots**

3) **Real, equal, rational roots**

4) **Real, unequal, rational roots**
Ex: Describe the Nature of the roots for the following equation.

\[ 2x^2 - 3x + 4 = 0 \]

\[ a = 2 \quad b = -3 \quad c = 4 \]

\[ b^2 - 4ac \]

\[ (-3)^2 - 4(2)(4) \]

\[ -23 \]

Roots are imaginary.
Ex: Parabola \( ax^2 + bx + c \ (x \neq 0) \) has two equal roots. What can be said about the appearance of the graph?

The parabola touches the axis at one point where the roots are equal. Discriminate equals 1 to zero.
Ex: Find the smallest possible integral value of $c$, such that $6x^2 - 4x + c = 0$ will have imaginary roots.

\[
\begin{align*}
\Delta &= b^2 - 4ac < 0 \\
(\color{green}{-4})^2 - 4(\color{green}{6})(c) &< 0 \\
16 - 24c &< 0 \\
24c &> 16 \\
\frac{24c}{24} &> \frac{16}{24} \\
\frac{2}{3} &< c
\end{align*}
\]
Ex: Find the nature of the roots for the following equations using the discriminant and verify the roots graphically using your graphing calculator.

\[ x^2 - 2x - 3 = 0 \]
\[ a = 1 \quad b^2 - 4ac = (-2)^2 - 4(1)(-3) = 16 \]
\[ Real, Rational, Not equal \]

\[ x^2 + 6x + 9 = 0 \]
\[ a = 1 \quad b^2 - 4ac = (6)^2 - 4(1)(9) = 0 \]
\[ Real, Rational, Equal \]

\[ x^2 - 2x + 8 = 0 \]
\[ a = 1 \quad b^2 - 4ac = (-2)^2 - 4(1)(8) = -28 \]
\[ Imaginary \]

\[ 2x^2 - 6x - 3 = 0 \]
\[ a = 2 \quad b^2 - 4ac = (-6)^2 - 4(2)(-3) = 60 \]
\[ Real, Irrational, Unequal \]
Homework is on pages 183 - 184 #1 - 20